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On the distinct phenomena of suffusion and suffosion

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Seepage-induced internal instability is a phenomenon whereby fine particles are transported from a non-plastic soil. A distinction can readily be made between a washed-out soil structure that remains intact and one in which some form of destruction or collapse of the structure accompanies the migration of fine particles. The three variables of a measured value of mass loss, a measured value of volume change and a value of change in hydraulic conductivity, deduced from measurements of hydraulic gradient and flow rate, are sufficient to quantify, and hence distinguish between, seepage-induced internal instability phenomena. The term 'suffusion' is advocated to describe the non-destructive response, which may be quantified by a mass loss, no change in volume and an increase in hydraulic conductivity. The term 'suffosion' is recommended to describe the instability phenomenon whereby the transport of fine particles by seepage flow is accompanied by a collapse of the soil structure. Accordingly, this distinct internal instability phenomenon may be quantified by a mass loss, a volumetric contraction and a change in hydraulic conductivity.

KEYWORDS: dams; erosion; fabric/structure of soils; seepage

NOTATION

i_{av}	hydraulic gradient
k	hydraulic conductivity
l	specimen length
m	mass
V	total volume
v	discharge velocity
v_s	seepage velocity

INTRODUCTION

Seepage-induced internal instability of soils used in the construction of earth dams has attracted increasing attention since the discovery of sinkholes at the Balderhead dam (Sherard, 1979; Vaughan & Soares, 1982) and at the Tarbela dam (Lowe, 1988), through to the more recent finding of sinkholes in the WAC Bennett dam (Stewart & Garner, 2000) and forensic assessment of the potential for internal instability in a database of earth dams (Rönnqvist, 2010). The current state of practice is limited to an empirical evaluation of the shape of the grain size distribution curve, for example (CDA, 2007; USBR, 2011), and companion materials testing given uncertainty in the empirical methods (ICOLD, 2014). A qualitative understanding of factors governing the onset of instability has informed efforts to model the response (Muir Wood *et al.*, 2010; Scholtes *et al.*, 2010; Bonelli & Marot, 2011) and firmly established that any advance towards a mechanics-based simulation requires a clear distinction between various instability phenomena. The objective of this contribution is to provide a conceptual framework for the characterisation of distinct seepage-induced internal instability phenomena.

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ON THE PHENOMENA OF SEEPAGE-INDUCED INTERNAL INSTABILITY

The first systematic experimental study of internal instability (USACE, 1953) sought to identify an optimum filter gradation, from permeameter testing of sand and gravel mixtures, in an investigation of 'inherent stability' (Table 1). Subsequent research has yielded some notable contributions, such as

- the underlying philosophical observations of Kezdi (1979) and Kovács (1981)
- the association of internal instability with sinkholes in dams (Sherard, 1979)
- empirical rules for assessing the potential for internal instability (Kenney & Lau, 1985, 1986; Burenkova, 1993; Wan & Fell, 2008)
- experimentally driven advances towards a mechanics-based framework (Skempton & Brogan, 1994; Li & Fannin, 2012).

There appears to be consensus in the literature that internal instability is a phenomenon whereby fine particles are transported from a non-plastic soil by seepage flow (Table 1). A subtle distinction has been proposed between the migration of particles within a soil and out of a soil (Kovács, 1981). A more significant distinction has been made between a washed-out soil structure that remains intact and one in which some form of destruction or collapse of the structure accompanies the seepage-induced migration of fine particles (Wittmann, 1978; Kezdi, 1979; Kovács, 1981; Åberg, 1993; Burenkova, 1993; Garner & Sobkowicz, 2002; Richards & Reddy, 2007; Li, 2008; Moffat *et al.*, 2011). Here, the term 'structure' is used to take into account both the soil fabric and its stability (Mitchell & Soga, 2005).

Various terms have been used to describe internal instability, with and without some form of collapse of the soil structure. More unfortunate is the use of the same term for each of these distinct phenomena: 'suffosion' has been used to describe both destructive (Burenkova, 1993; Garner & Sobkowicz, 2002; Richards & Reddy, 2007; Moffat *et al.*, 2011) and non-destructive phenomena (Wittmann, 1978; Molenkamp *et al.*, 1979; Burenkova,

Table 1. Descriptions of seepage-induced internal instability phenomena

Author	Phenomenon	Description
USACE (1953)	Inherent stability	'Inherent stability was determined by the degree to which the gradation curves of the sections compared with those of the original material'
Wittmann (1978)	Suffosion	'…mixtures with a gravel skeleton (…) permit transport of sand particles into, inside and out of the skeleton. These phenomena are known as suffosion (transport out of the skeleton) and colmatation or sluicing (transport into the skeleton). These processes influence the permeability of the mixture… The skeleton 'remains stable'
	Erosion	'For pure sand or only small amounts of the gravel component this [deformation] phenomenon is known as piping by heave, whereas mixtures with a higher amount of coarser particles show the phenomenon of erosion piping. Erosion leads to collapse of the mixture, with the finer particles drawn away by the seeping water'
Kezdi (1979)	Suffusion	'Suffusion is a phenomenon where water, while seeping through the pores, carries along the fine particles without destroying the soil structure'
	Erosion	'Erosion…destroys the soil structure. Not only are single grains or fractions disrupted, but the whole soil structure is progressively destroyed and tubelike cavities are formed'
Molenkamp *et al.* (1979)	Suffosion	'…migration of the finer material of the very filter layer…'
	Internal instability	'The internal stability of the filter material could possibly be checked by comparing the grain size distributions of the top and bottom part of the column'
Sherard (1979)	Internal erosion stability	'…a concentrated leak developed through the core which caused a type of internal erosion in which the soil fines are eroded selectively and carried out of the core, leaving the coarse sand and gravel particles behind to act as a pervious drain. The volume of fine material eroded was larger than the volume of the void spaces between the coarser soil particles causing progressive collapse of the material above the initial leakage channel, which action finally reaches the dam surface as manifested by the sinkhole or crater'
Jones (1981)	Suffosion	'To Pavlov (1898) and Savarensky (1940)…suffosion meant mechanical removal of loose particles… Both mechanical and chemical forms of suffosion process were described by Russian, Polish and French workers…'
Kovács (1981)	Suffusion	'Redistribution of fine grains within the layer…when the solid volume of the layer is not changed only the local permeability is altered'
	Internal suffusion	'…when the solid volume of the layer is not changed only the local permeability is altered'
	External suffusion	'…when the volume of the solid matrix is reduced, accompanied by an increase in permeability'
	Destruction of the skeleton	'Subsidence of the layer, when some of the coarse grains are removed from the solid matrix, and thus the load of overlying layers causes the total volume of the layer to decrease'
Kenney & Lau (1985)	Internal or grading instability	'…refer to the ability of a granular material to prevent loss of its own small particles due to disturbing agents such as seepage and vibration'
	Suffosion	'…the transport of small particles from a soil…'
den Adel *et al.* (1988)	Internal instability	'Because of its wide grain size distribution, the small grains can easily be washed out, through the skeleton of the large grains'
Lafleur *et al.* (1989)	Internally unstable	'…substantial migration took place within the layers… This trend was supported by the permeability curves…'
Åberg (1993)	Grading instability	'…the material has loose grains, which are so small that they can pass through the constrictions between fixed grains…loose grains should be considered potentially unstable'
	Internal filter formation process or self-filtration	'…when this process proceeds in a…upward direction…shrinkage of the material during washout causes movement in the overlying material…and thereby also make washout of fixed grains possible'
Burenkova (1993)	Inner suffosion	'Inner suffosion takes place, when fine particles are transported in the soil structure… This process can lead to an increase of the permeability of the soil…'
	Outer suffosion	'Outer suffosion means transportation of fine fractions out of the soil… This leads to an increase of the void ratio, the permeability and…to instability of the wh[o]le structure'
	Internal instability	'The tests showed particle movements along the interface of the test apparatus and the soil specimen under seepage flow. The readings of the piezometers altered during the tests and particles eroded'
Skempton & Brogan (1994)	Internal instability	'…the sand can migrate within the interstices of a framework of primary fabric formed predominantly of the gravel particles and can be washed out…'
	Segregation piping	'…failure took the form of piping of the fines whereas the gravel particles remained practically undisturbed… In the stable materials this occurred at approximately the critical gradient given by piping theory (…), but in the unstable materials migration and strong piping of fines took place at gradients of about one fifth to one third of the theoretical value…'
	Onset of instability	'A further increase in gradient causes a disproportionate increase in flow, leading towards failure either by piping…or by the opening of a horizontal crack…which then works its way upwards until piping occurs throughout'
Chapuis *et al.* (1996)	Suffossion	'…migration of fine particles of a soil within its own pore space'
	Internal instability	If a 0–20 mm base has an internal instability problem…creation of small layers with high capillarity and low permeability…are likely to develop with time'
	Internal segregation	'The internal segregation of particles was evaluated by wet sieve analysis…' and '…changes [in permeability] appeared with [internally unstable] gradation 2'

Table 1. Continued

Author	Phenomenon	Description
Chapuis & Aubertin (2004)	Suffusion	'…a permeating process, often a fluid movement towards a surface or over a surface; thus, using it for internal erosion would be incorrect…'
	Suffosion	'…this word is not found in English and French dictionaries'
	Suffosion	'…correctly represents the phenomenon of internal erosion. It is unfortunate that it has been forgotten after having been used a long time ago by military engineers as an undermining technique'
Honjo *et al.* (1996)	Self-filtration	'As flow starts, the finer base particles pass through the filter but the coarser particles are caught at the base soil–filter interface quickly forming a thin dense layer [which] consists of the coarser particles. The rest of the base soil is then protected by this layer…'
	Stable	'The loss of base soil is observed for a limited time and the development of [a] self-filtration layer is prominent as observed from the after test sieve analysis'
	Unstable	'Loss of base soil is observed to be continuous and the whole sample is subject to disturbance'; 'self-filtration layer is absent in this case', also related to 'progressive settlement'
Garner & Sobkowicz (2002)	Suffosion	'…the mass movement of the fine fraction within the skeleton of a dispersed, potentially unstable coarse fraction', associated with an increase of permeability
	Suffusion	'…the redistribution of fine grains within a stable densely packed skeleton' and associated reduction in permeability, referring to Kovács (1981) 'Internal suffusion'
Wan & Fell (2004)	Suffusion or internal instability	'…an internal erosion process which involves selective erosion of fine particles from the matrix of a soil made up of coarse particles'
	Unstable	'…a change in the grain-size distribution of the test sample after the test'; '…signs of erosion…are observed as the hydraulic gradient across the test sample increases'; 'The colour of the effluent provided an indication of whether or not erosion was taking place…'
	Stable	'…the grain-size distribution of the test sample will remain unchanged after the test'
Richards & Reddy (2007)	Suffosion	'This process can result in a loose framework of granular material with relatively high seepage flows that leads to collapse of the soil skeleton (McCook 2004). In non-cohesive materials suffosion leads to zones of high permeability (and water transmission), potential outbreaks of increased seepage, increased erosive forces and potential collapse of the skeletal soil structure.'
	Suffusion	'Gradual loss of finer matrix materials in a soil supported by a coarser grained skeleton is termed suffusion, which may lead to a more general collapse and loss of soil structure, termed suffosion (Kezdi 1979; McCook 2004)'
Fell & Fry (2007)	Suffusion or internal instability	'…selective erosion of fine particles from the matrix of coarser particles…by seepage flow, leaving behind an intact soil skeleton formed by the coarser particles'
Li (2008)	Internal suffosion	'…refers to the redistribution of finer particles within layers that is accompanied by a change in local hydraulic conductivity'; 'No displacement or mass loss was observed'
	External suffosion	'…refers to the scouring of finer particles that is associated with an overall increase in hydraulic conductivity'; 'A contractive displacement and mass loss [was] observed'
Bendahmane *et al.* (2008)	Suffusion	'…the permeability…decreased by a factor of 10 during the tests where erosion was initiated…characterised by some diffuse mass losses'
Bonelli & Marot (2011)	'Suffusion (or suffosion)'	'…an internal erosion process by which finer soil particles are detached from the solid matrix and transported through pore constrictions by seepage flow'
Moffat *et al.* (2011)	Suffusion	'…the finer fraction of an internally unstable soil moves within the coarser fraction without any loss of matrix integrity or change in total volume…' 'The visual observations, and the companion spatial and temporal variations of local hydraulic gradient, reveal a transport of finer particles from the soil with each increment of hydraulic gradient that yields a relatively slow and small change in local permeability, but no change in volume'
	Suffosion	'…particle migration yields a reduction in total volume and a consequent potential for collapse of the soil matrix (Richards and Reddy 2007)' 'Visual observations and the companion spatial and temporal variations of local hydraulic gradient, reveal a particle loss that yields a relatively large and rapid change in local permeability and a companion change in specimen volume'

1993) of seepage-induced internal instability (Table 1). In contrast, 'suffusion' has only been used to describe the non-destructive phenomenon of seepage-induced internal instability. The lack of clarity arising from use of the same term to describe these two distinct phenomena is an impediment to scientific progress.

EXPERIMENTAL EVIDENCE OF INTERNAL INSTABILITY PHENOMENA

Laboratory experiments have yielded advances in the understanding of seepage-induced internal instability. Early investigations relied solely on a change in the grain size distribution as an indicator of internal instability (USACE, 1953; Molenkamp *et al.*, 1979; Kenney & Lau, 1985). Mass loss is commonly used to quantify internal instability – the loss may be characterised directly by collection of soil eroded from the specimen (e.g. den Adel *et al.*, 1988; Åberg, 1993; Bendahmane *et al.*, 2008), or indirectly from a change in gradation of the test specimen (e.g. Chapuis *et al.*, 1996; Wan & Fell, 2004; Moffat *et al.*, 2011). On occasion, both methods have been used (Lafleur *et al.*, 1989; Honjo *et al.*, 1996; Li, 2008). Kovács (1981) first explicitly postulated that a change in local permeability accompanies the mass loss of fine particles, something to which the United States Army Corps of Engineers had earlier alluded (USACE, 1953). Lafleur

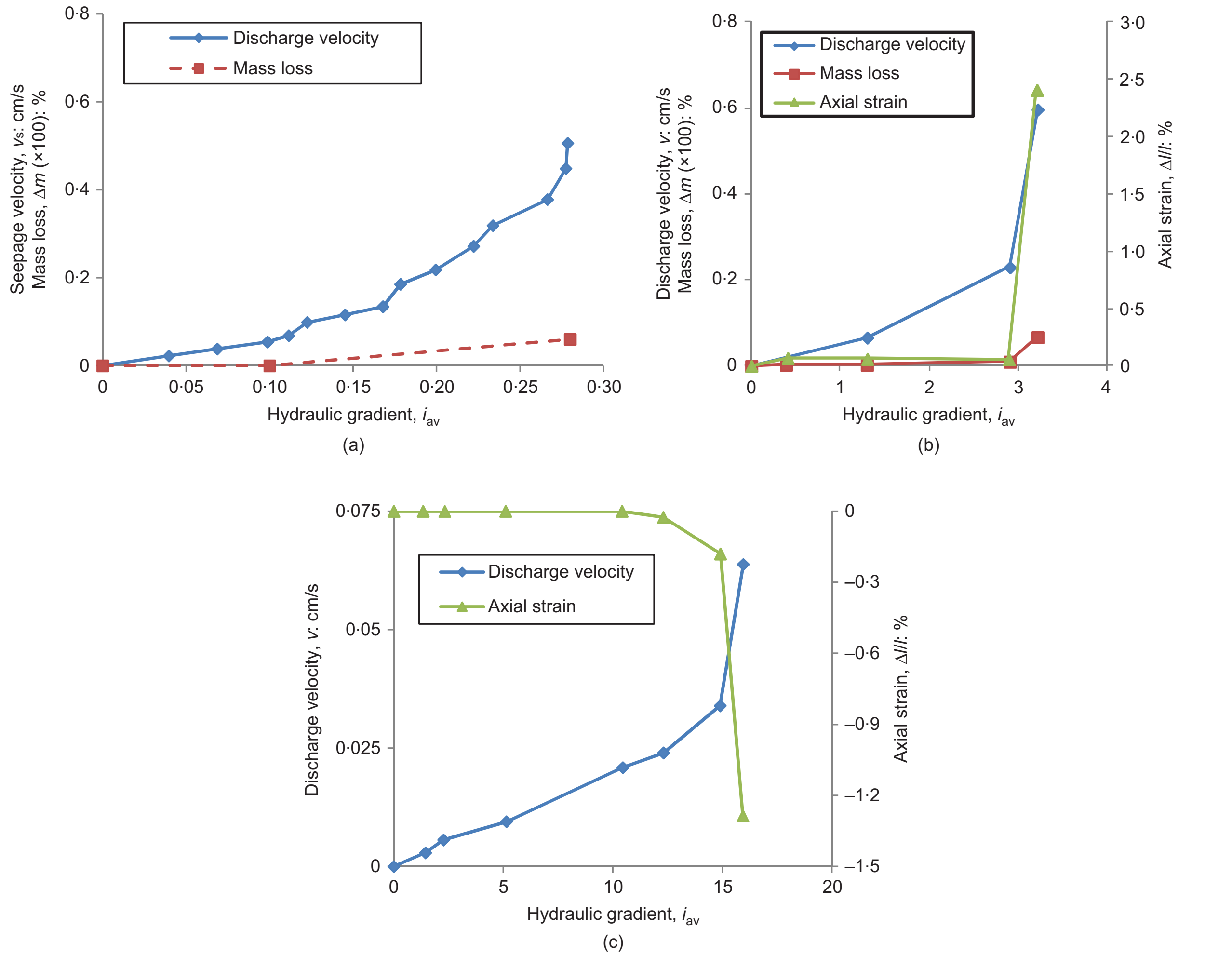

Fig. 1. (a) Non-destructive seepage-induced internal instability (after Skempton & Brogan (1994)). (b) Destructive seepage-induced internal instability (after Li (2008)). (c) Seepage-induced instability (after Li (2008))

et al. (1989) first used temporal and spatial changes in hydraulic conductivity, deduced from measurements of hydraulic gradient and flow rate, as indicators of an internally unstable material. Although measurements of hydraulic gradient and flow rate are commonly recorded (Skempton & Brogan, 1994; Chapuis *et al.*, 1996; Garner & Sobkowicz, 2002; Wan & Fell, 2004; Li, 2008; Moffat *et al.*, 2011), it appears that changes in hydraulic conductivity are seldom reported in a systematic manner. Notably, Skempton & Brogan (1994), and later Li (2008), respectively, used measurements of hydraulic gradient and seepage and discharge velocity to quantify the onset of instability.

Despite an early appreciation for the possible rearrangement of soil structure in response to the onset of internal instability (Kezdi, 1979; Kovács, 1981), measurement of deformation has not received the same widespread recognition that is given to measurements of mass loss, hydraulic gradient and flow rate. Only Honjo *et al.* (1996), Li (2008) and Moffat *et al.* (2011), in studies using a rigid-wall permeameter, report systematic measurements of axial deformation associated with seepage-induced internal instability. The recent development of a flexible-wall permeameter for the investigation of internal instability has enabled the measurement of volumetric deformation (Bendahmane *et al.*, 2008; Chang & Zhang, 2011).

Experimental evidence suggests that the three variables of: (i) a measured value of mass loss; (ii) a measured value of volume change; and (iii) a value of change in hydraulic conductivity, deduced from measurements of hydraulic gradient and flow rate, are sufficient to quantify, and hence distinguish between, seepage-induced internal instability phenomena:

- Skempton & Brogan (1994) reported seepage-induced internal instability in a specimen of gap-graded sandy gravel subject to upward flow in a rigid-wall permeameter. The top surface of the specimen was unloaded. The linear relation between hydraulic gradient (i_{av}) and seepage velocity (v_s) to $i_{av} \leq 0{\cdot}11$ (Fig. 1(a)) indicates a constant value of hydraulic conductivity. At $i_{av} > 0{\cdot}11$, a disproportionate increase of seepage velocity with hydraulic gradient indicates an increase in hydraulic conductivity. At $i_{av} = 0{\cdot}2$, there was 'strong general piping of fines throughout', but '[t]he gravel particles remain undisturbed'. The experimental findings were mass loss, no collapse of the soil structure and hence no volume change, and an increase in hydraulic conductivity.
- Li (2008) reported seepage-induced internal instability in a specimen of gap-graded glass beads subject to downward flow in a rigid-wall permeameter. The top surface of the specimen was loaded. The linear relation between hydraulic gradient and discharge velocity to $i_{av} \leq 2{\cdot}9$ (Fig. 1(b)) indicates a constant value of hydraulic conductivity. At $i_{av} > 2{\cdot}9$, a disproportionate increase of discharge velocity with hydraulic gradient indicates an increase in hydraulic conductivity. According to Li (2008), 'Negligible mass loss and axial displacement were observed during these flow stages… Upon imposing a

small increase in hydraulic gradient to $i_{av} = 3{\cdot}2$, a modest amount of finer particles (6·9%) was lost from the specimen… A total downward axial displacement of 2·5 mm was measured, resulting in an axial strain of 2·6%'. Thus, in contrast to Skempton & Brogan's findings, the experimental findings of Li were mass loss, contractive volume change associated with collapse of the soil structure, and an increase in hydraulic conductivity.

In the classic description of 'piping by heave' of sand, Terzaghi & Peck (1948) observe that 'This process greatly increases the permeability of the sand (…) and [t]he surface of the sand then rises'. Accordingly, this seepage-induced instability phenomenon may be similarly quantified by the same three variables:

- Li (2008) reported seepage-induced instability of a specimen of widely graded sand and gravel subject to upward seepage flow in a rigid-wall permeameter. The top surface of the specimen was loaded. The linear relation between hydraulic gradient and discharge velocity to $i_{av} \leq 15$ (Fig. 1(c)) indicates a constant value of hydraulic conductivity. Li notes that 'Upon imposing an increase in hydraulic gradient to $i_{av} = 16{\cdot}0$ (…) a large upward displacement of 4·2 mm was measured, resulting in a strain of 1·3%'. The experimental findings were no mass loss, dilative volume change and an increase in hydraulic conductivity.

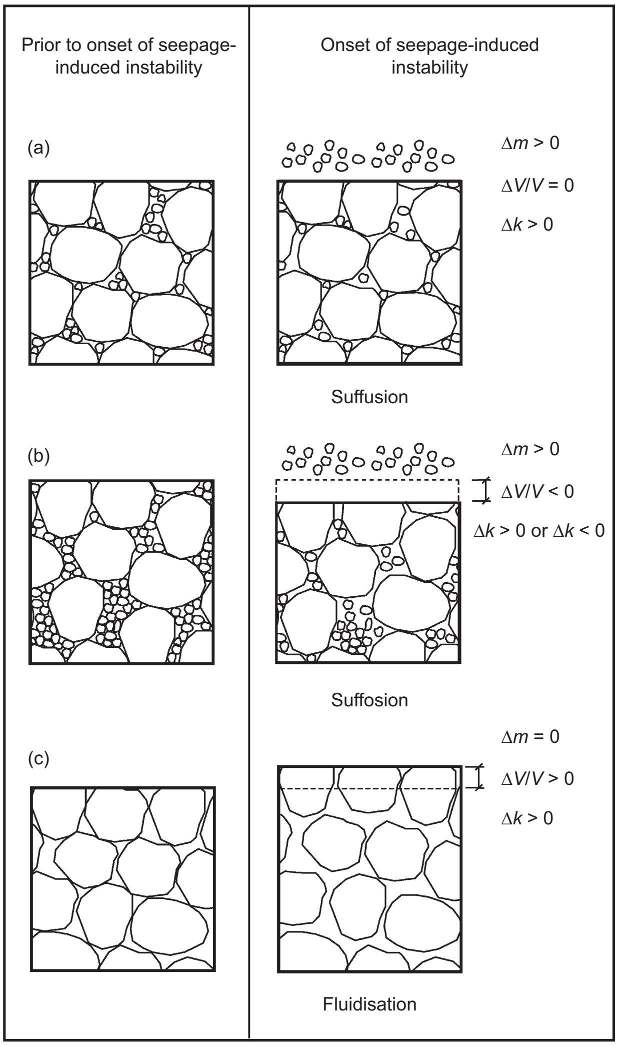

Fig. 2. Schematic illustration of seepage-induced instability phenomena: (a) suffusion; (b) suffosion; (c) fluidisation

CONCEPTUAL FRAMEWORK

In an internally unstable soil, the phenomenon whereby fine particles are transported by seepage flow and the soil structure remains intact may be quantified by a mass loss, no change in volume and an increase in hydraulic conductivity (Fig. 2(a)). The term 'suffusion' has been commonly used to describe this non-destructive response (Table 1) and its continued use is therefore advocated. In contrast, the internal instability phenomenon in which the transport of fine particles by seepage flow is accompanied by a collapse of the soil structure may be quantified by a mass loss, a volumetric contraction and a change in hydraulic conductivity (Fig. 2(b)). It is a different response, and one with potential for unacceptable deformation, for which the term 'suffosion' is recommended.

An internally stable soil may be either uniformly graded or broadly graded. In the presence of upward seepage flow, the instability phenomenon may be quantified by a volumetric expansion, accompanied by an increase in hydraulic conductivity (see, for example, Fig. 2(c)). The term 'fluidisation' has been used to describe this response (Vardoulakis, 2004) and its continued use appears appropriate.

The recommendation to distinguish between the three phenomena of suffusion, suffosion and fluidisation, based on mass loss, volume change and change in hydraulic conductivity, is illustrated in Fig. 3 in a revised version of the Venn diagram originally proposed by Garner & Fannin (2010). The conceptual framework presented here can thus successfully distinguish between the non-destructive and destructive seepage-induced internal instability phenomena of suffusion and suffosion, respectively, in both a qualitative and a quantitative manner. A benefit of distinguishing between these two phenomena in engineering practice arises where the risk of suffusion might be managed differently to the risk of suffosion, for example in dam engineering.

CONCLUDING REMARKS

Advances in understanding the mechanics of seepage-induced internal instability and managing the associated risk that it presents to geotechnical infrastructure are contingent on a clear distinction being made between instability phenomena. A conceptual framework is developed here such that a distinction can be reasonably made between phenomenological responses based on mass loss and volume

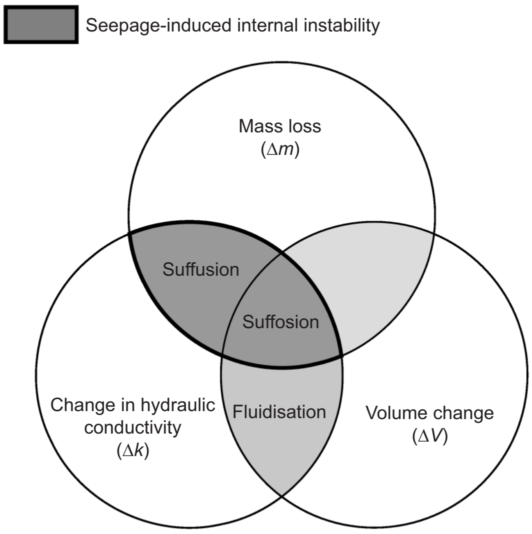

Fig. 3. Conceptual framework for seepage-induced instability phenomena

change, which can be measured directly, and change in hydraulic conductivity, which can be deduced from measurement of hydraulic gradient and flow rate.

Recognising the important distinction between non-destructive and destructive phenomena in engineering practice, it is recommended that

- suffusion be characterised as seepage-induced mass loss without change in volume, accompanied by an increase of hydraulic conductivity
- suffosion be characterised as seepage-induced mass loss accompanied by a reduction in volume and a change in hydraulic conductivity
- fluidisation be characterised as seepage-induced volumetric expansion accompanied by an increase in hydraulic conductivity, with no mass loss.

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